Axial segregation of granules in a horizontally rotating drum

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Summary

Generally, mixing granular materials can lead to segregation, though we need to blend multiple granules in industries.

In this paper the phenomenon of axial segregation in a horizontally rotating drum is discussed. Axial segregation means that two granules make bands along the axis of rotation and they move and merge one after another as time goes on.

First brief explanation and history of the phenomenon is shown, followed by explanation of the widespread model “dynamic angle of repose”. In the second place, two experimental results are presented in order to argue that the model is not sufficient especially when we consider band merging. Finally some discussion points for the future are proposed.

Introduction

When you want to mix two granular materials, it seems very reasonable way to let them rotate in a drum mixer. However it doesn’t make homogeneous state if those materials fulfill requirements such as difference in diameter, density and so on. Rather they segregate first in a rotating plain, which is called radial segregation and then make bands along the rotating axis, which merge as the drum continues rotating (axial segregation). The former can be imagined, but the latter is beyond our expectation and very fascinating (Fig. 1).

This very counterintuitive phenomenon is first reported by Oyama in 1939. Then Donald and Rosenman argued in 1962 as follows: the condition for banding is that the static angle of repose of the small species is larger than that of the larger particles. In contrast, Das Gupta et al. explored the parameter space of the experiment and concluded in 1991 that it’s so-called “dynamic angle of repose” which determines whether the segregation would be radial or axial. This angle is the slope which the granular material makes in steady-

1This brief history refers to Levine [1].
state rotation. It’s poorly defined, however, since the configuration of a flowing granule is not straight at all (Fig. 2).

A granule whose angle is smaller won’t go up along the wall of a drum as high as that whose angle is larger, which means that the former starts rolling down earlier. Moreover, the former tends to roll down the slope easily. Thus this smaller-angle granule gathers at lower places, magnifying initial unevenness of particle concentration. This is a simple explanation of the “dynamic angle of repose” model.

![Fig. 1 Example of axial segregation](image1.png)

![Fig. 2 Dynamic angles of repose](image2.png)

White sand is smaller and has larger angle, resulting in intermittent flow and more complicated configuration.
Savage writes that axial segregation is modeled by diffusion equation with negative diffusion coefficient if the drift due to difference in dynamic angles of repose exceeds the effect of Fickian diffusive flux [2].

Of course, diffusion equation with negative coefficient diverges, but Kobayashi et al. managed to use following equations in simulating the model and got a feasible result shown in Fig. 3 [3].

Taking x-axis along the rotation axis, let \( u(x, t) \) and \( v(x, t) \), concentration of smaller granule at \( x \), height of the top end of avalanching free surface at \( x \), respectively;

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \mu \frac{\partial}{\partial x} \left[ u(1 - u) \frac{\partial v}{\partial x} \right].
\]

\[
0 = l^2 \frac{\partial^2 v}{\partial x^2} + u - v.
\]

The first equation determines how concentration \( u \) changes according to configuration \( v \). On the other hand, \( v \) is got from \( u \) with the second equation.

This model surely reproduces the result of experiment to some extent\(^2\).

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\(^2\)Actually there are some points which doesn't match experimental results I got.

One is that there are ceasing points of a black band and a white band at the same time. One species of granules should have smaller angle of repose and a band of it won't be stopped by a band of the other species.

Another is that bands are too stable at the latter part of the simulation.
100% filling should disturb axial segregation because the avalanching surface disappears. However two kinds of sand did segregate, which seemed to have been big discovery at first, only to turn out false.

There was actually tiny avalanching surface because of compression of two granules which was done after starting rotation. In certifying experiment, I repeated two processes as follows. Rotate the drum a couple of times and add both red and white sand to the full. The result was no axial segregation.

Interesting point is inside the drum. Red sand, which is ten times as big as white, dominates the surface, but white gets into a region near surface (Fig. 4).

Then the reason why tiny avalanching surface causes axial segregation is due to strong radial segregation. In that case, even small rearrangement of granules can reveal the white band inside.

From this experiment, it’s said that the dynamic angle of repose model is valid in band generation, but it’s not sufficient. We have to consider other features such as mechanism below the surface\(^3\).

Second result of experiment is about the bottom part of avalanching surface. Although the dynamic angle of repose model assumes that the bottom is flat, the real configuration is not flat at all (Fig. 5).

Fig. 4 cross section of 100 % experiment

The white band is higher and red lower. In addition, it is asymmetric. We can see some white sand being caught in red low band. It should have something to do with band moving and merging.

\(^3\)Although I can’t confirm whether there is some band along the rotation axis beneath surface in case of 50 % filling, some authors mention existence of the band using MRI technique\(^5, 6\).
This bottom part should determine the direction of band moving, which is not considered in one-dimensional simple models of dynamic angle of repose.

**Discussion**

Here there discussion points are proposed.

In this experimental condition white sand should cross red bands in order that band merging is complete. The existence of white band core must be considered. Sub-surface flow which is shown in Fig. 6 also seems contributing.

Secondly, ‘how does a band determine the direction to move in?’

Especially when two red bands have merged, there should be white sand inside them. Then the new band rearranges granules inside and determines the next direction. It should feel some pressure from surrounding bands: how far and how strong it functions? At the same time, arrangement of granules is following very local mechanism as shown in Fig. 7.

Finally, It’s almost impossible to reproduce critical characters of axial segregation by one-dimensional models.

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4This period is called metastable states by Hill *et al.* [5].
It’s obvious that band merging involves sand catching at the bottom of free surface. Over simplifying model can’t hold both band creation and band merging. A new model which includes property of the bottom of free surface is desired though I’ve not succeeded in constructing a model like that yet.

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References